The First Law of Systems: Conservation of Complexity

SWISSED 2018

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Airbus A320neo

A320neo offers up to ***20 per cent savings in fuel burn*** per seat by 2020, two tonnes of additional payload, ***500 nautical miles of more range***, lower operating costs, along with a nearly ***50 per cent reduction in engine noise and NOx emissions 50 per cent below*** the current industry standard.

Source: https://www.airbus.com/aircraft/passenger-aircraft/a320-family/a320neo.html

SYS Journal 20th Anniversary

IDSS
MIT INSTITUTE FOR DATA, SYSTEMS, AND SOCIETY

- **Has Systems Engineering progressed at all in the last 20 years?**
- **What is the theoretical (scientific) basis of Systems Engineering?**
- **What is the First Law of Systems Engineering?**

SYSTEMS ENGINEERING

MAY 2018 | VOLUME 21 | NUMBER 3

WILEY

SYSTEMS ENGINEERING

The Journal of **The International Council on Systems Engineering**

20th Anniversary Special Issue

IN THIS ISSUE
Systems engineering 20th anniversary special issue
A historical perspective on systems engineering
Evolution of systems engineering scholarship from 2000 to 2015, with particular emphasis on software
Model-based systems engineering: Motivation, current status, and research opportunities
Sensitivity analysis methods for mitigating uncertainty in engineering system design
Guiding the behavior of sociotechnical systems: The role of agent-based modeling
System architecting and design space characterization
Evidence-based systems engineering
Building the tower without climbing it: Progress in engineering systems

INCOSE

MAY 2018
VOLUME 21 | NUMBER 3
ISSN 1098-1241

WILEY

15/05/18 2:46 PM

First Laws in Science

- First Law of Mechanics
  - Conservation of Momentum
  - Isaac Newton 1687

$$\frac{d}{dt}(p_1 + p_2) = 0.$$

- First Law of Thermodynamics
  - Conservation of Energy
  - Rudolf Clausius 1850

$$\Delta U = Q - W.$$

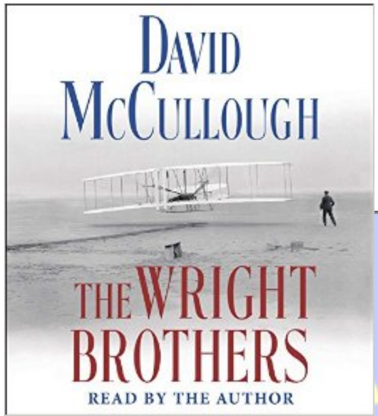
- ***What is the conserved quantity in Systems?***

COMPLEXITY

Why should we care about complexity?

How do we quantify complexity?

The First Law of Systems Engineering ?



The Wright Flyer

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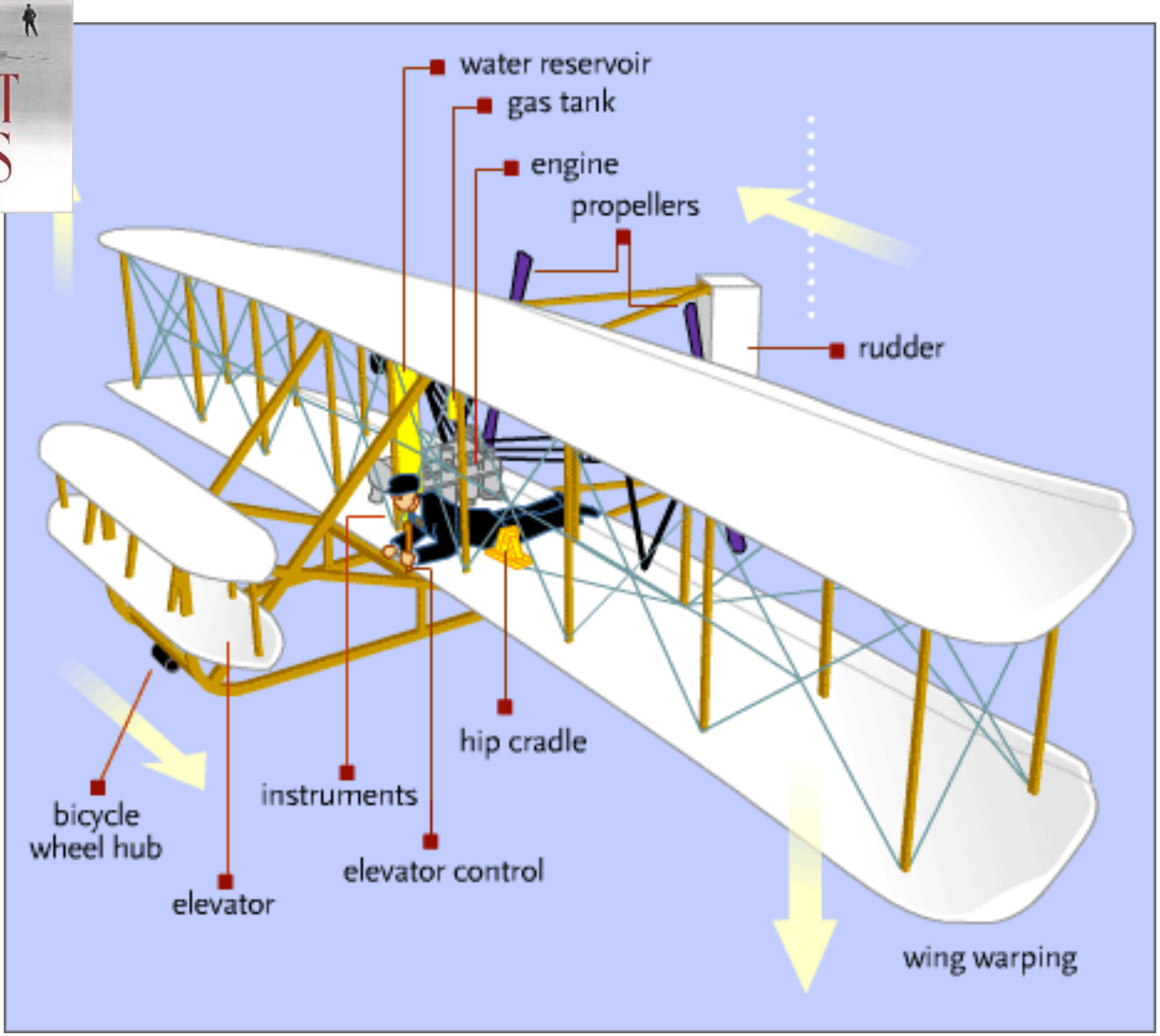
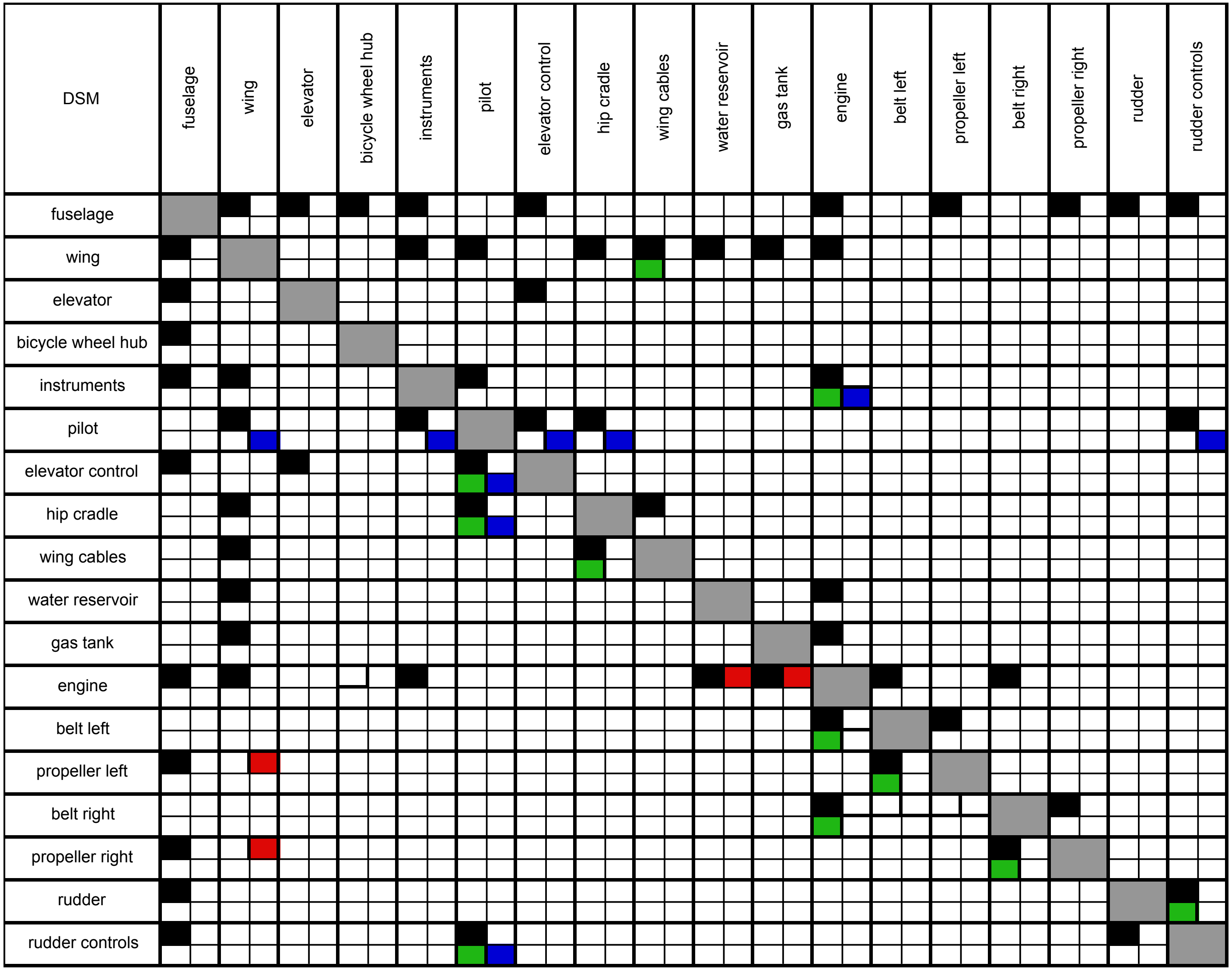


Structural DSM of Wright Flyer



DSM	fuselage	wing	elevator	bicycle wheel hub	instruments	pilot	elevator control	hip cradle	wing cables	water reservoir	gas tank	engine	belt left	propeller left	belt right	propeller right	rudder	rudder controls
fuselage	▨	■	■	■	■		■					■		■		■	■	■
wing	■	▨			■	■		■	■ 🟩	■	■	■						
elevator	■		▨				■											
bicycle wheel hub	■			▨														
instruments	■	■			▨	■						■ 🟩 🟦						
pilot		■ 🟦			■ 🟦	▨	■ 🟦	■ 🟦										■ 🟦
elevator control	■		■			■ 🟩 🟦	▨											
hip cradle		■				■ 🟩 🟦		▨	■									
wing cables		■						■ 🟩	▨									
water reservoir		■								▨		■						
gas tank		■									▨	■						
engine	■	■			■					■ 🟥	■ 🟥	▨	■		■			
belt left												■ 🟩	▨	■				
propeller left	■	🟥											■ 🟩	▨				
belt right												■ 🟩			▨	■		
propeller right	■	🟥													■ 🟩	▨		
rudder	■																▨	■ 🟩
rudder controls	■					■ 🟩 🟦											■	▨

Legend	
■	Physical connection
🟥	Mass flow
🟩	Energy flow
🟦	Information flow

DSM 18x18

Connections

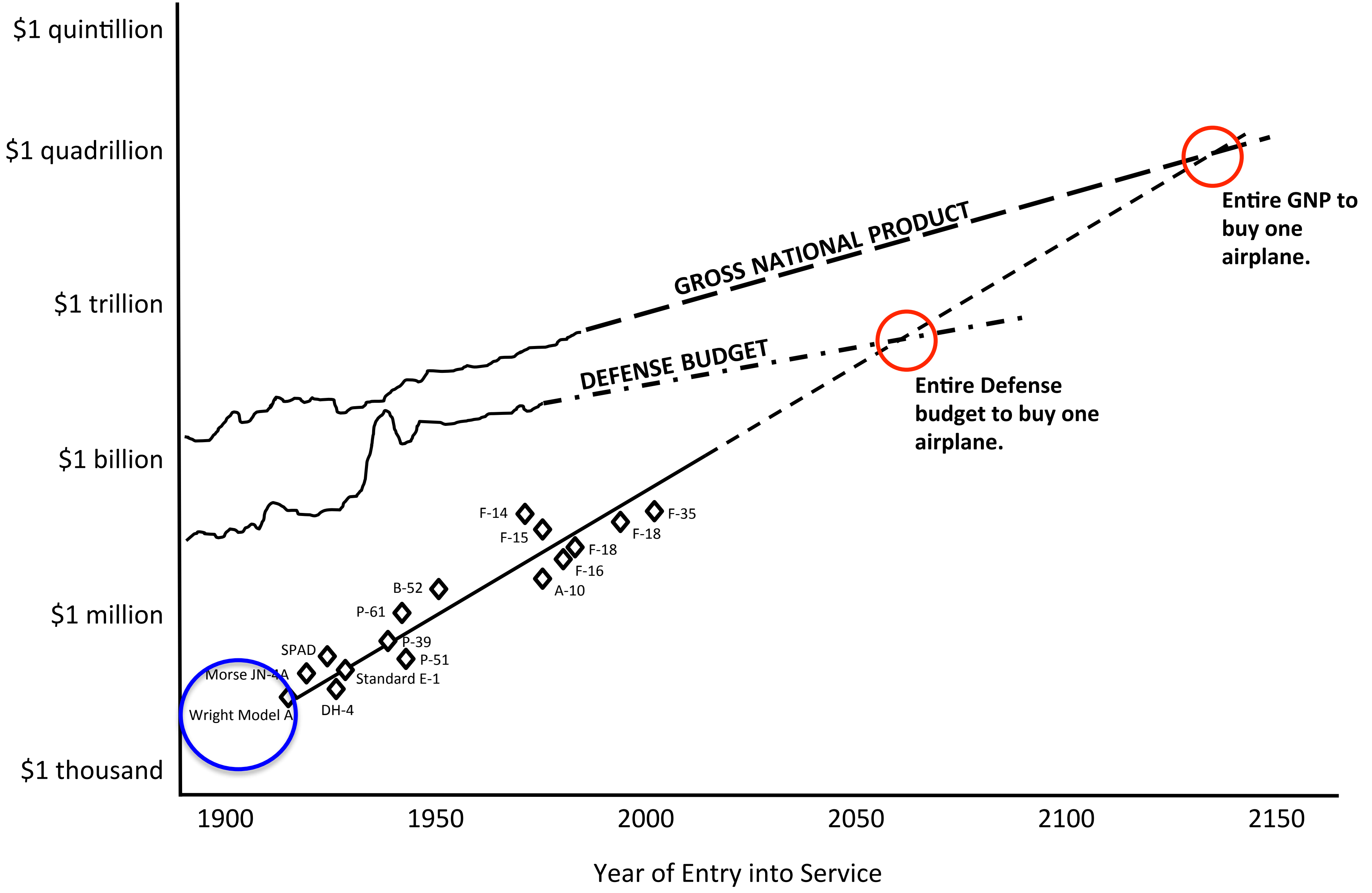
62 Physical
4 Mass Flow
11 Energy Flow
9 Info Flow
Total: 86

NZF = 86/1,224
= **7% density**

<k>=~5

Design Structure Matrix (DSM) – captures structure of elements of form

Augustine's 16th Law

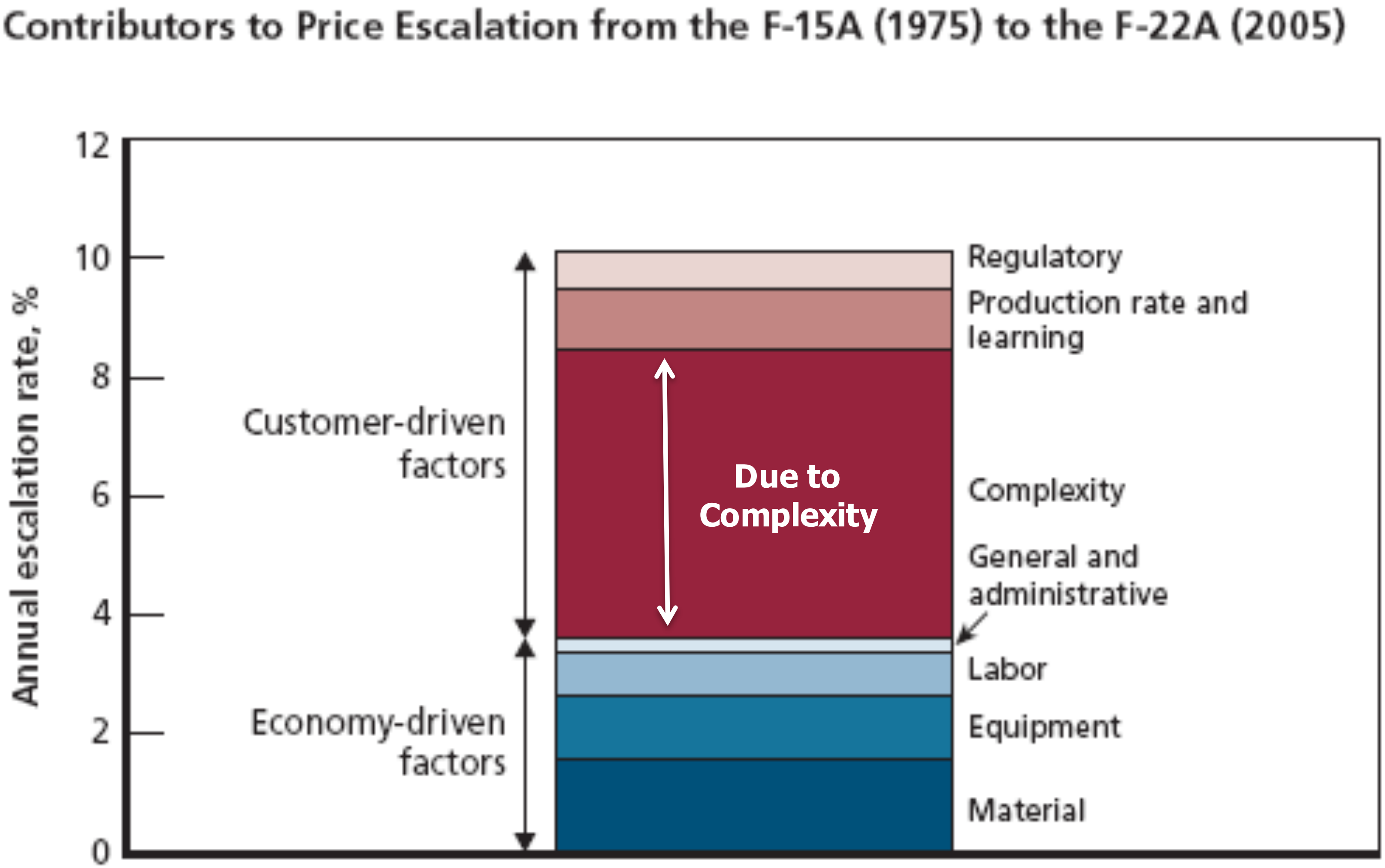


Entire GNP to buy one airplane.

Entire Defense budget to buy one airplane.

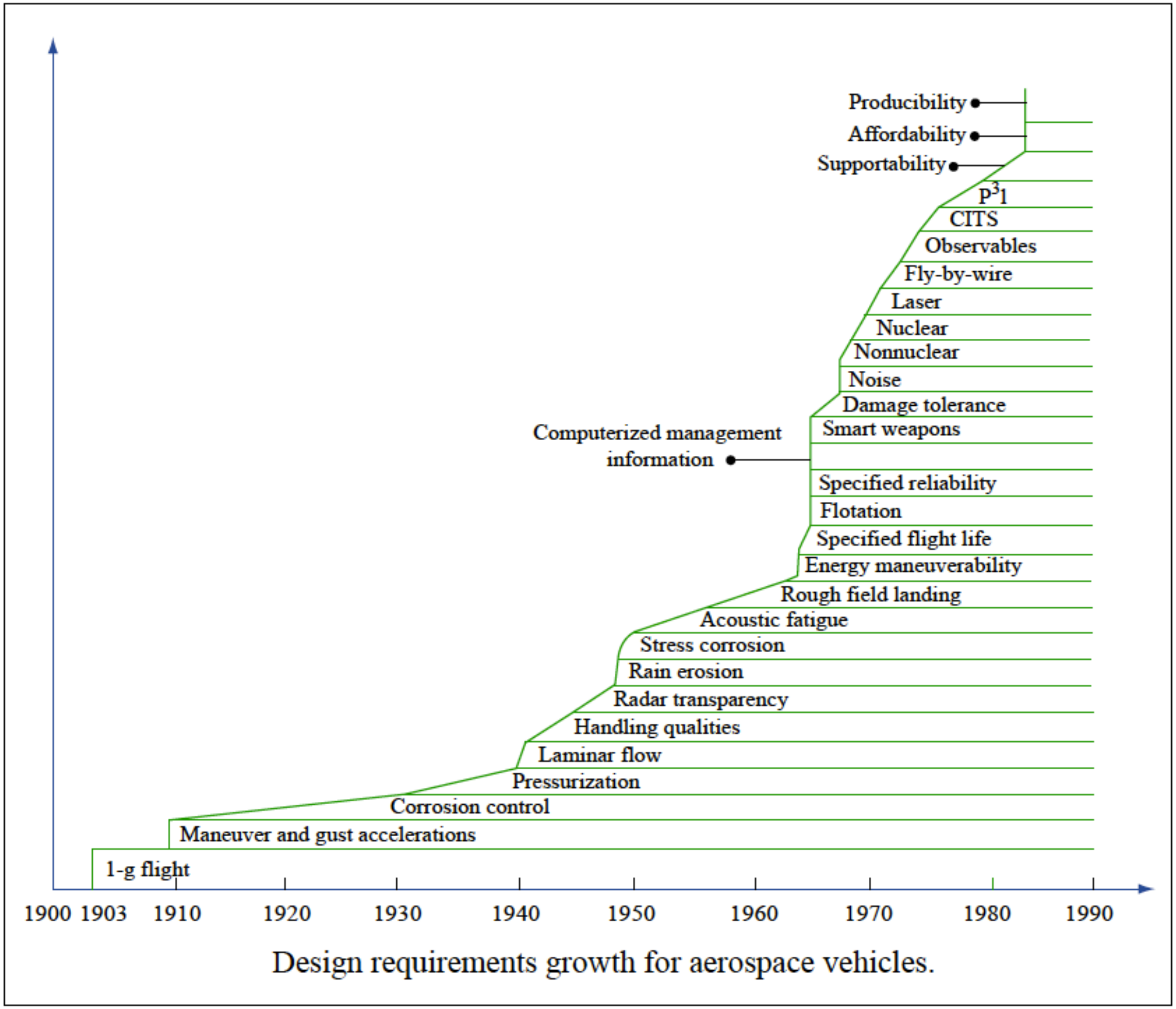
Norm Augustine, *Augustine's Laws*, 6th Edition, AIAA Press, 1997.

What is driving this escalation of cost?



Source: DARPA TTO (2008)

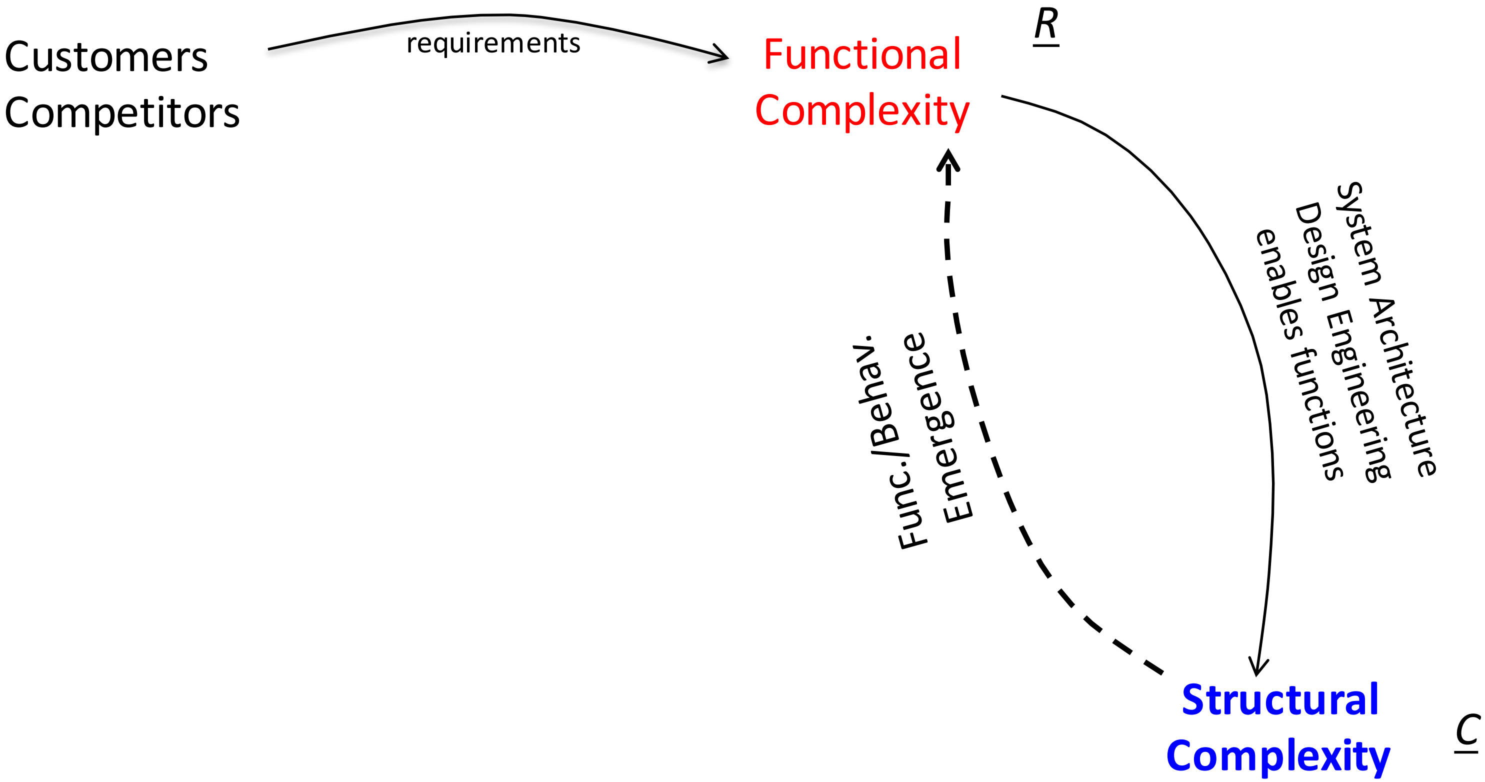
Complexity Driven by Functional Requirements Explosion



Design requirements growth for aerospace vehicles.

Image by MIT OpenCourseWare.

Two Dimensions of Complexity



Why should we care about complexity?

How do we quantify complexity?

The First Law of Systems Engineering ?

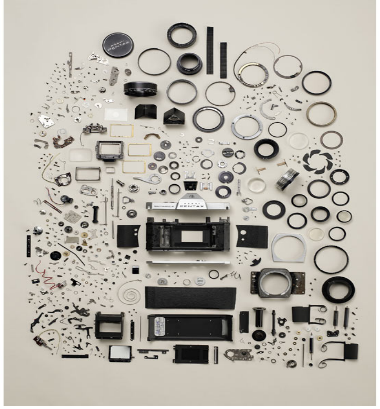
The Structural Complexity Metric

Structural Complexity, $C = C_1 + C_2 . C_3$

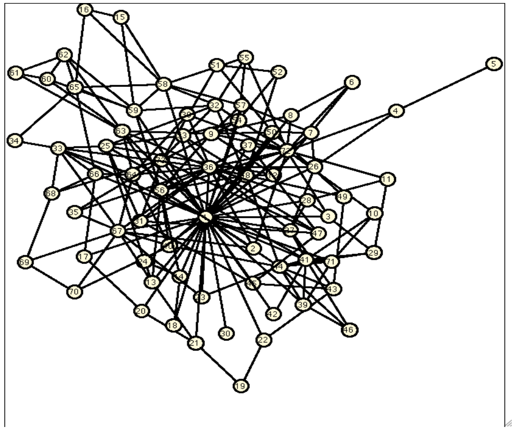
[This functional form inspired by the solution of the *steady-state Schrodinger equation* of organic molecular systems [Gutman 1978, 2000].]

Complexity due to topological formation (a scaling factor) – due to dependency structure

Complexity due to components alone (number and heterogeneity of components)

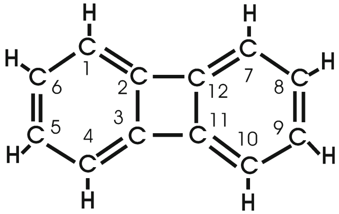


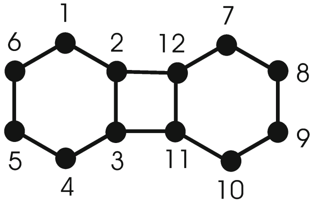
Complexity due to pair-wise component interactions (number and heterogeneity of interactions)



Sinha, Kaushik, and Olivier L. de Weck. "A network-based structural complexity metric for engineered complex systems." In *Systems Conference (SysCon), 2013 IEEE International*, pp. 426-430. IEEE, 2013.

System Hamiltonian and Complexity





$$[\mathbf{H}]_{ij} = \begin{cases} \alpha & \text{if } i = j \\ \beta & \text{if the atoms } i \text{ and } j \text{ are chemically bonded} \\ 0 & \text{if there is no chemical bond between the atoms } i \text{ and } j. \end{cases}$$

$$\mathbf{H} = \alpha\begin{bmatrix} 1&0&0&0&0&0&0&0&0&0&0&0 \\ 0&1&0&0&0&0&0&0&0&0&0&0 \\ 0&0&1&0&0&0&0&0&0&0&0&0 \\ 0&0&0&1&0&0&0&0&0&0&0&0 \\ 0&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&0 \\ 0&0&0&0&0&0&0&0&1&0&0&0 \\ 0&0&0&0&0&0&0&0&0&1&0&0 \\ 0&0&0&0&0&0&0&0&0&0&1&0 \\ 0&0&0&0&0&0&0&0&0&0&0&1 \end{bmatrix} + \beta\begin{bmatrix} 0&1&0&0&0&1&0&0&0&0&0&0 \\ 1&0&1&0&0&0&0&0&0&0&0&1 \\ 0&1&0&1&0&0&0&0&0&0&1&0 \\ 0&0&1&0&1&0&0&0&0&0&0&0 \\ 0&0&0&1&0&1&0&0&0&0&0&0 \\ 1&0&0&0&1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1&0&0&0&1 \\ 0&0&0&0&0&0&1&0&1&0&0&0 \\ 0&0&0&0&0&0&0&1&0&1&0&0 \\ 0&0&0&0&0&0&0&0&1&0&1&0 \\ 0&0&1&0&0&0&0&0&0&1&0&1 \\ 0&1&0&0&0&0&1&0&0&0&1&0 \end{bmatrix}.$$

$$H = \alpha I_n + \beta A(G)$$

$$H\psi = \varepsilon\psi$$

$$|\varepsilon_i| = \alpha + \beta\sigma_i;\ \varepsilon_\pi = \sum_{i=1}^{n} h_i|\varepsilon_i|$$

$$\varepsilon_\pi = n\alpha + \beta\sum_{i=1}^{n} h_i\sigma_i \le n\alpha + \beta\underbrace{\left(\sum_{i=1}^{n} h_i\right)}_{n}\underbrace{\left(\sum_{i=1}^{n}\sigma_i\right)}_{E(A)}$$

$$\therefore \varepsilon_\pi \le n\alpha + n^2\beta\left(\frac{E(A)}{n}\right)$$

Introduce the notion of *configuration energy*:

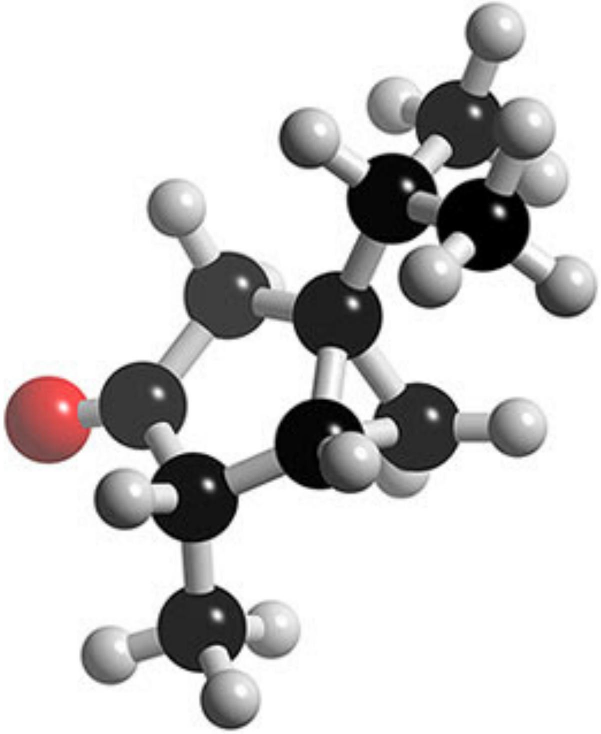
$$\Xi := \underbrace{n\hat{\alpha}}_{C_1} + \underbrace{m\hat{\beta}}_{C_2}\underbrace{\left(\frac{E(A)}{n}\right)}_{C_3} = C_1 + C_2C_3$$

Use the above functional form to measure the complexity associated to the system structure – ***Structural Complexity*** of the system where α's stand for component complexity while β's stand for interface complexity:

$$C = C_1 + C_2C_3$$

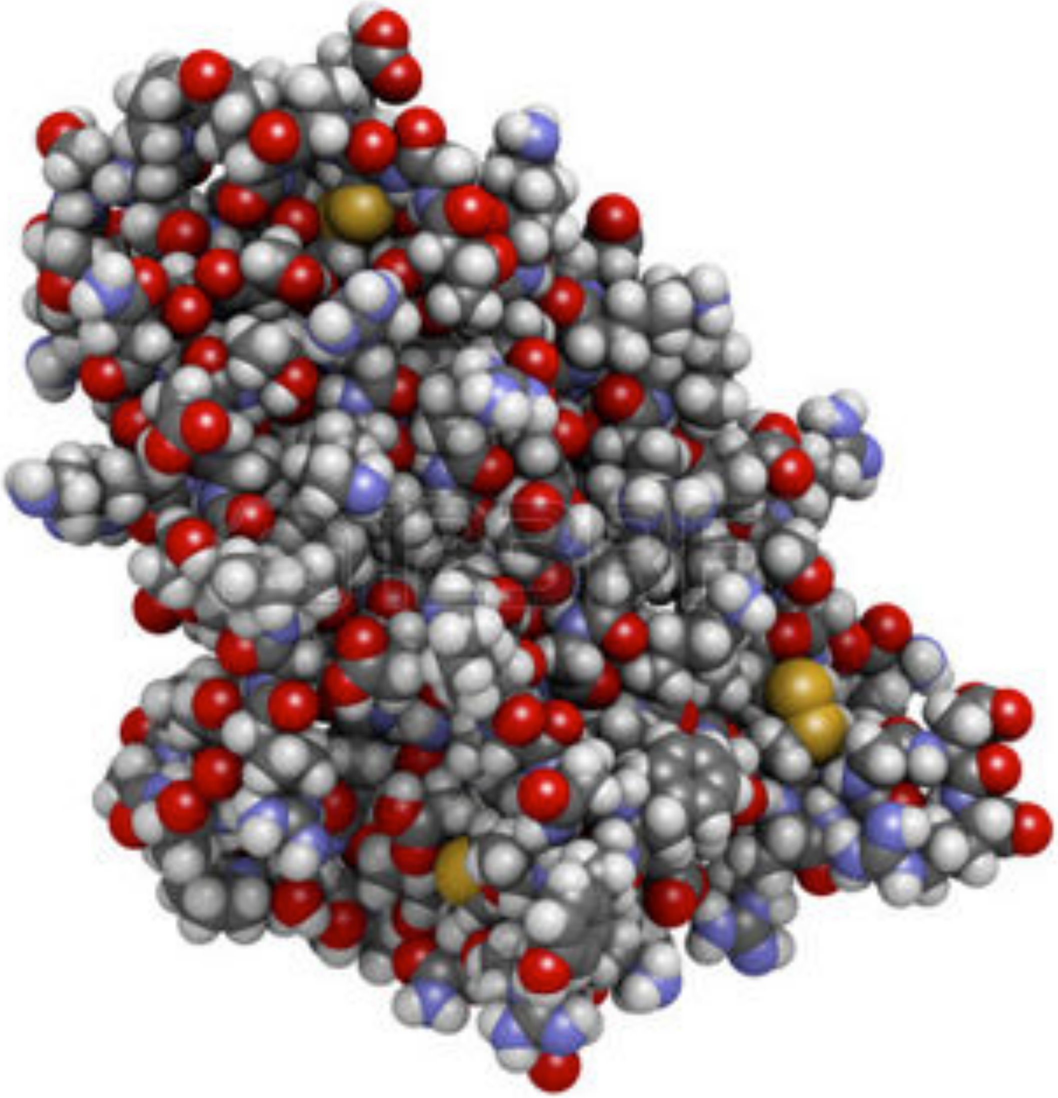
$$= \sum_{i=1}^{n}\alpha_i + \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\right)\left(\frac{E(A)}{n}\right) = \sum_{i=1}^{n}\alpha_i + \left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}\right)\gamma E(A)$$

Simple Molecule



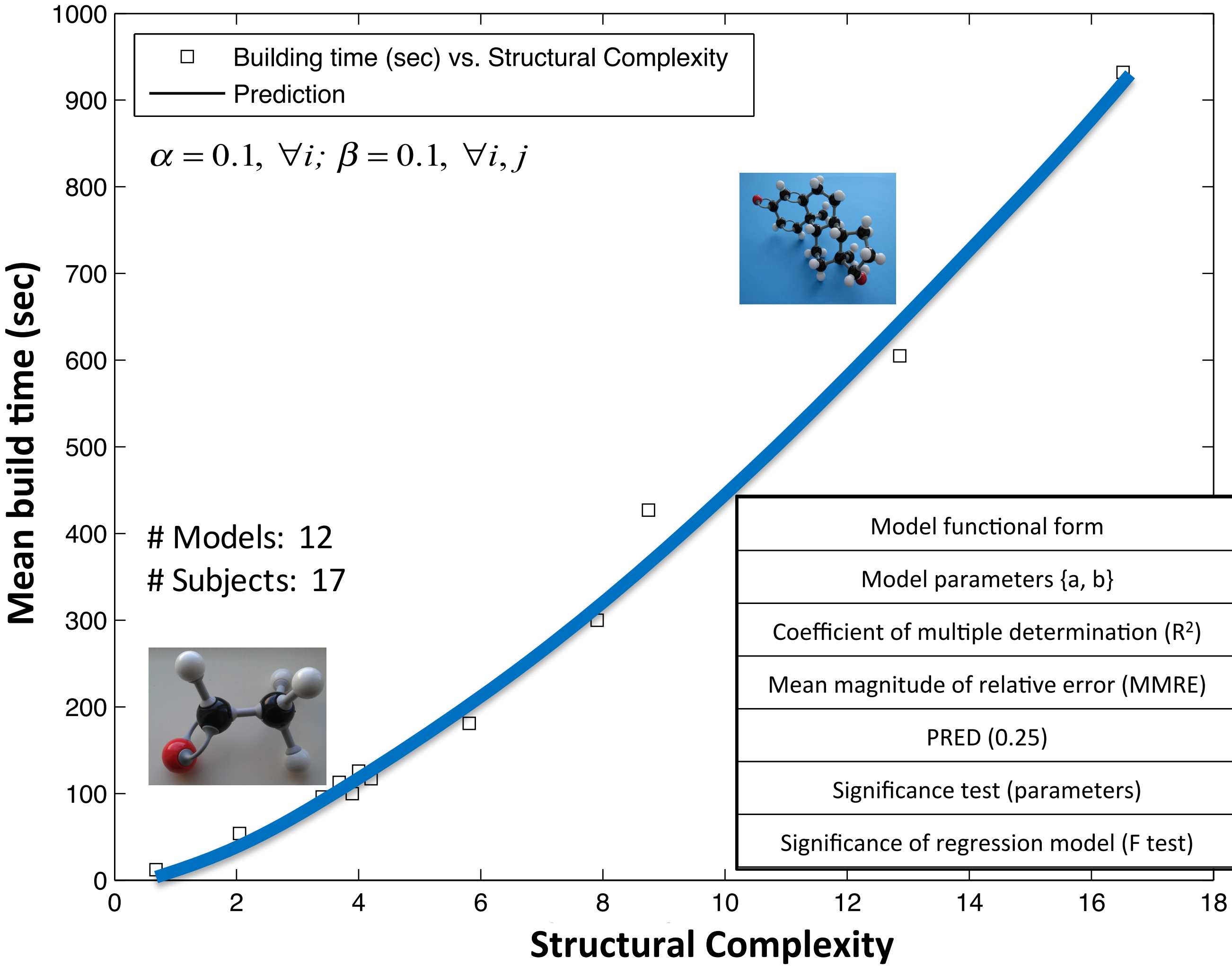
Molecule #10

Complex Molecule



https://en.wikipedia.org/wiki/Erythropoietin

Experimental results are super-linear



Model functional form	$Y = aX^b$
Model parameters {a, b}	{ 14.68, 1.4775}
Coefficient of multiple determination (R^2)	0.992
Mean magnitude of relative error (MMRE)	0.107
PRED (0.25)	0.9167
Significance test (parameters)	$t_a = 28.2$, $t_b = 30.67$ ($>t_0 = 2.131$)
Significance of regression model (F test)	$f = 124 > f_{0.05,1,10} = 4.54$

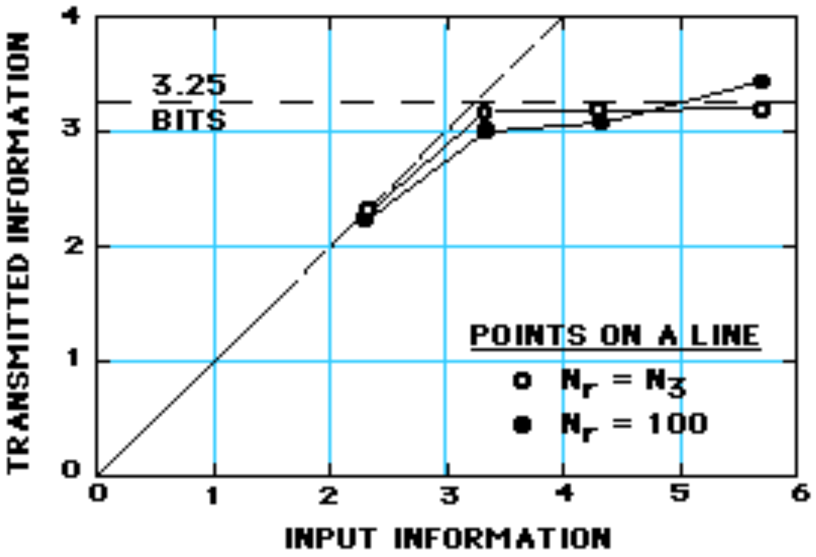
$$\textit{Structural Complexity},\ C = O(n^{1.08}) \leftarrow \textit{mild super-linearity}$$

$$\textit{Average build time},\ t = O(C^{1.48}) \leftarrow \textit{strong super-linearity}$$

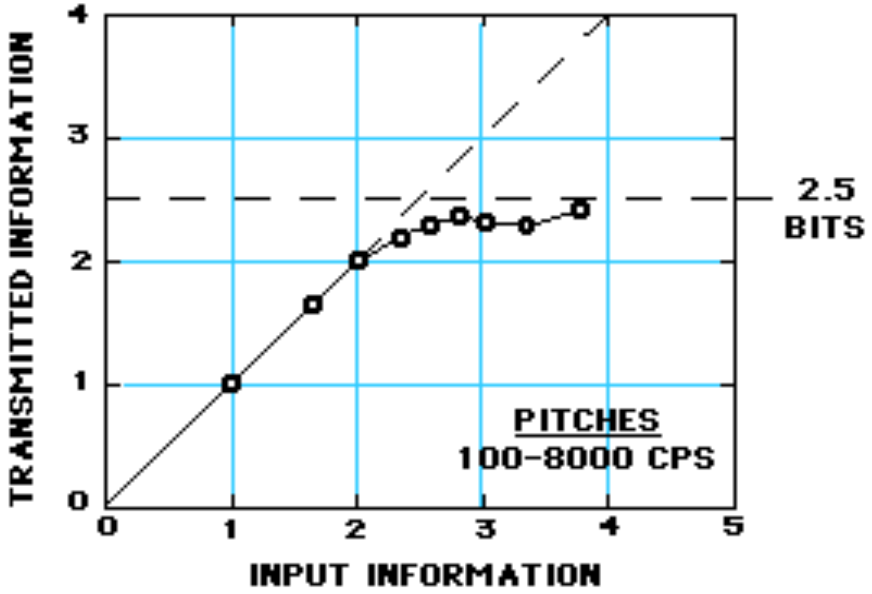
Magic Number 7+/-2

- Human Cognitive Limits for Processing Information
- George Miller (1956)
- http://www.musanim.com/miller1956/

Position of a Pointer
on Linear Interval



Auditory Pitch
Experiments

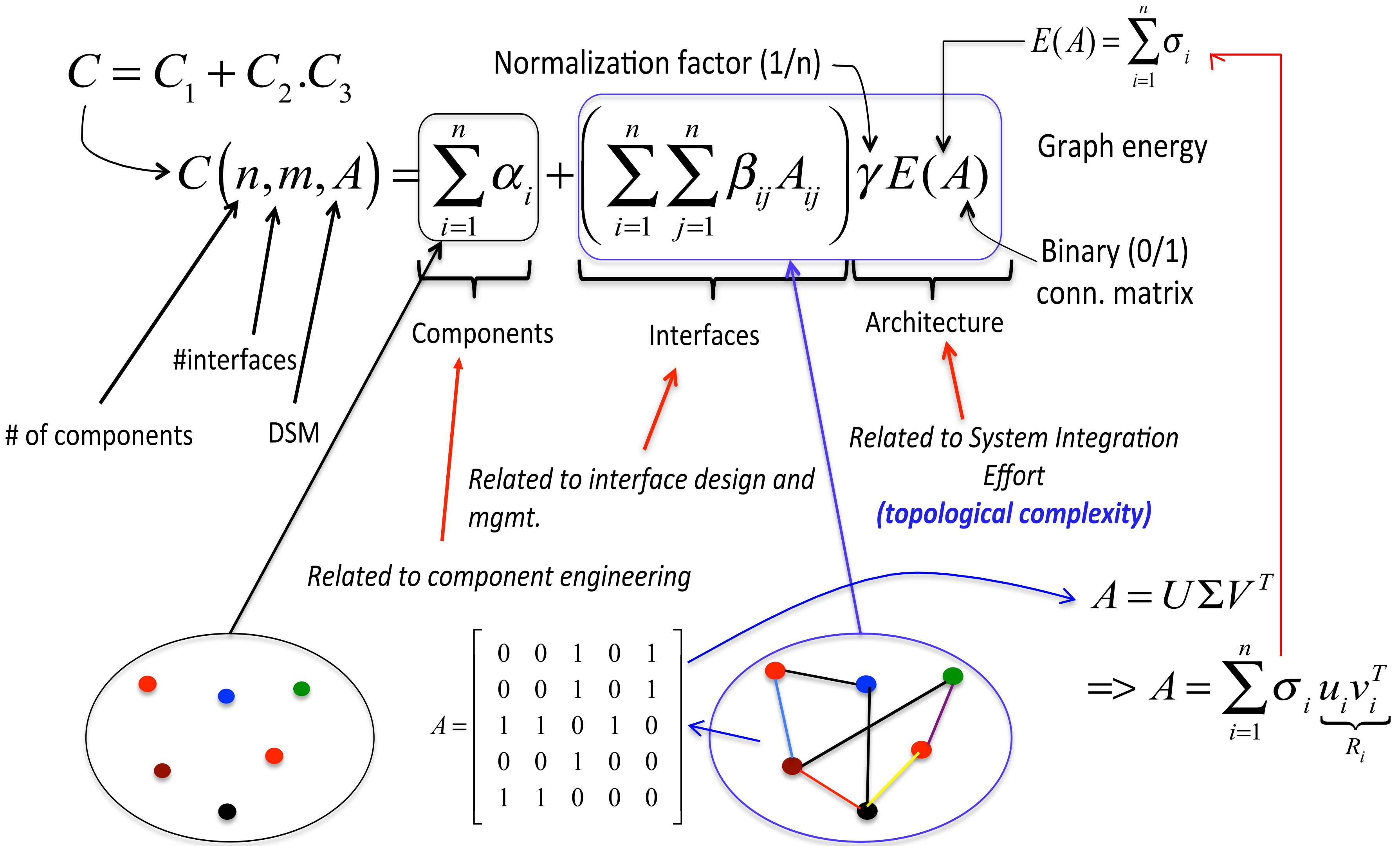


Metric Validity: Weyuker’s Criteria

- Graph Energy stands out as both computable and satisfies Weyuker’s criteria (1998) and establishes itself as a theoretically valid measure (i.e., construct validity) of complexity.

Complexity Measure	**Computability**	**Aspect emphasized**	**Weyuker’s Criteria**
Number of components [Bralla, 1986]	✔	Component development (count-based measure)	✗
Number of interactions [Pahl and Beitz, 1996]	✔	Interface development (count-based measure)	✗
Whitney Index [Whitney *et al.*, 1999]	✔	Components and interface developments	✗
Number of loops, and their distribution []	✗	Feedback effects	✗
Nesting depth [Kerimeyer and Lindemann, 2011]	✗	Extent of hierarchy	✗
Graph Planarity [Kortler *et al.*, 2009]	✔	Information transfer efficiency	✗
CoBRA Complexity Index [Bearden, 2000]	✔	Empirical correlation in similar systems	✗
Automorphism-based Entropic Measures [Dehmer *et al.*, 2009]	✗	Heterogeneity of network structure, graph reconfigurability	✔
Matrix Energy / Graph Energy	✔	Graph Reconstructabality	✔

Structural Complexity Metric



$$C = C_1 + C_2 . C_3$$

$$C(n,m,A) = \underbrace{\boxed{\sum_{i=1}^{n}\alpha_i}}_{\text{Components}} + \boxed{\underbrace{\left(\sum_{i=1}^{n}\sum_{j=1}^{n}\beta_{ij}A_{ij}\right)}_{\text{Interfaces}}\underbrace{\gamma E(A)}_{\text{Architecture}}}$$

Normalization factor (1/n)

$E(A) = \sum_{i=1}^{n}\sigma_i$

Graph energy

Binary (0/1) conn. matrix

# of components

#interfaces

DSM

Related to component engineering

Related to interface design and mgmt.

Related to System Integration Effort

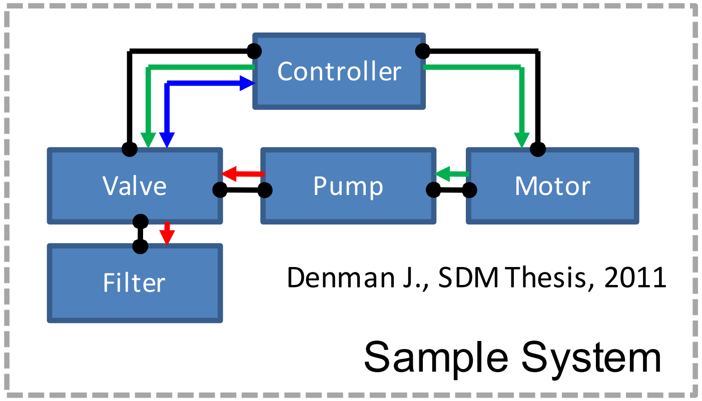
(topological complexity)

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$A = U\Sigma V^T$$

$$\Rightarrow A = \sum_{i=1}^{n}\sigma_i \underbrace{u_i v_i^T}_{R_i}$$

Example: Cyber-Physical System



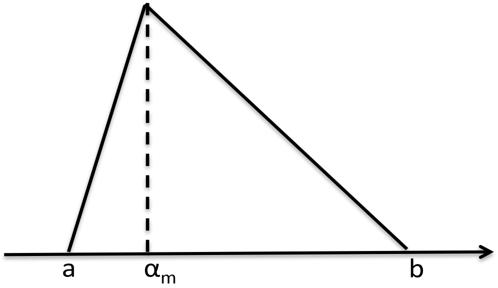
Denman J., SDM Thesis, 2011

Sample System

$$\xrightarrow{\text{aggregation}} \quad A = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Component	**ID**	**Complexity**
Controller	1	1.5
Pump	2	1.0
Valve	3	0.3
Filter	4	0.3
Motor	5	1.2

Comp. 1	Comp. 2	$1/c^{(k)}$
1	3	0.05
1	3	0.10
1	3	0.15
1	5	0.05
1	5	0.10
2	3	0.05
		0.10
		0.05
		0.15
		0.05
		0.10



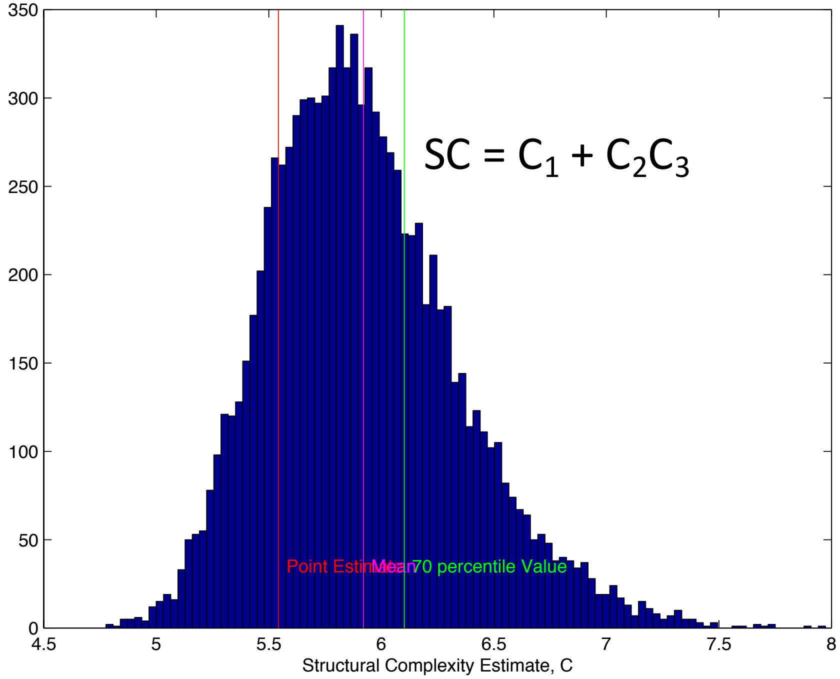
$$(b-\alpha_m) = p(\alpha_m - a)$$

$$p \in [1.0;3.0]$$
$$a \in [0.8\alpha_m;0.9\alpha_m]$$
$$b \in [1.1\alpha_m;1.6\alpha_m]$$

$$\beta_{ij}^{(k)} = g(\alpha_i,\alpha_j,c^{(k)})$$

$$\beta_{ij}^{(k)} = \frac{\max(\alpha_i,\alpha_j)}{c^{(k)}},$$

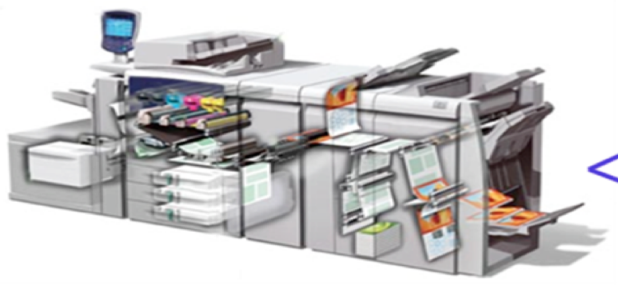
$$\forall \alpha_i,\alpha_j \neq 0,\ k \text{ is the interface type}$$



$$SC = C_1 + C_2C_3$$

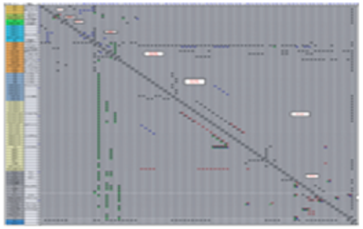
Structural Complexity Estimate, C

Complexity should be abstraction-Invariant



Digital Printing Press (Xerox) Example

Size: 50x50



Size: 91x91

DSM attribute	Coarse Representation	Finer representation
System size, N	50	91
C_3	1.3534	1.3597

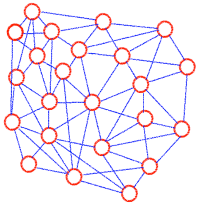
Functional Area	Coarse DSM (50x50)	Fine DSM (91x91)
ROS Assembly	4	10
Marking elements	16	38
Paper Path	7	12

Why should we care about complexity?

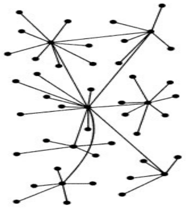
How do we quantify complexity?

The First Law of Systems Engineering?

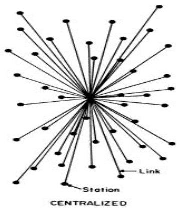
Topological Complexity: Important Properties



“Distributed” Architecture



“Hierarchical” Architecture



Centralized architecture

Simple components / constituents / building blocks with intricate connectivity structure

Higher system integration effort

Increasing Topological Complexity (C_3)

Complex components / constituents / building blocks with simple connectivity structure

Lower system integration effort

$$\textit{Centralized Architecture} \rightarrow \textit{hypoenergetic}, \ C_3 < 1$$

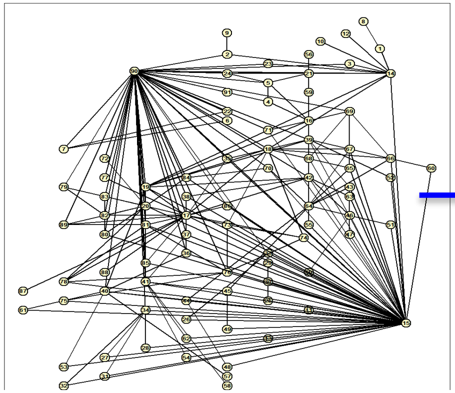
$$\textit{Hierarchical / layered Architecture} \rightarrow \textit{transitional}, \ 1 \le C_3 < 2$$

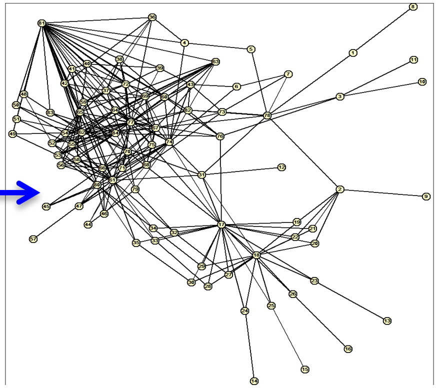
$$\textit{Distributed Architecture} \rightarrow \textit{hyperenergetic}, \ C_3 \ge 2$$

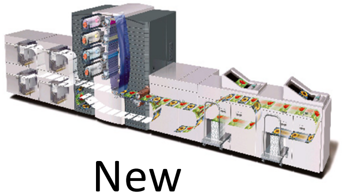
Case Study 1: Printing Engines

Old

Complexity = 186







New

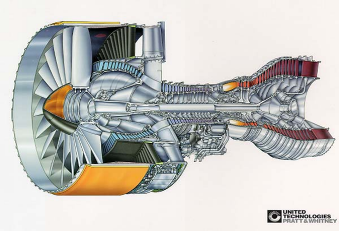
Complexity = 354

Complexity increase +90%

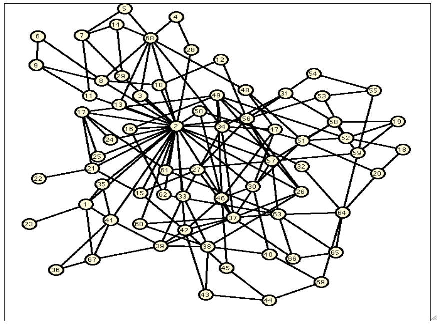
	C_1		C_2		C_3		C		C_{New} $/C_{Old}$
	Old	**New**	**Old**	**New**	**Old**	**New**	**Old**	**New**	
Most Likely	110.2	169	55.68	102.78	1.36	1.804	185.93	354.42	1.9062
Mean	125.62	213.6	63.29	130.6	1.36	1.804	211.69	449.2	2.122
Median	124.47	211.84	62.46	128.62	1.36	1.804	209.42	443.88	2.12
70 percentile	127	219	65.82	134.2	1.36	1.804	216.2	461.1	2.133

- Trend towards more distributed architecture with higher structural complexity and significantly higher development cost*

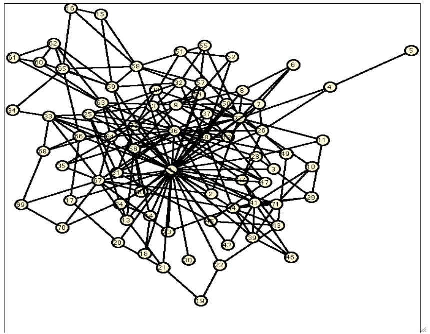
Case Study 2: Aircraft Engines

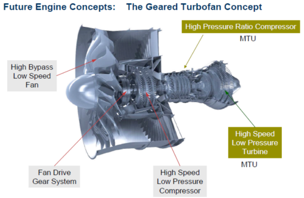


Old



Complexity = 351





New

Complexity = 499

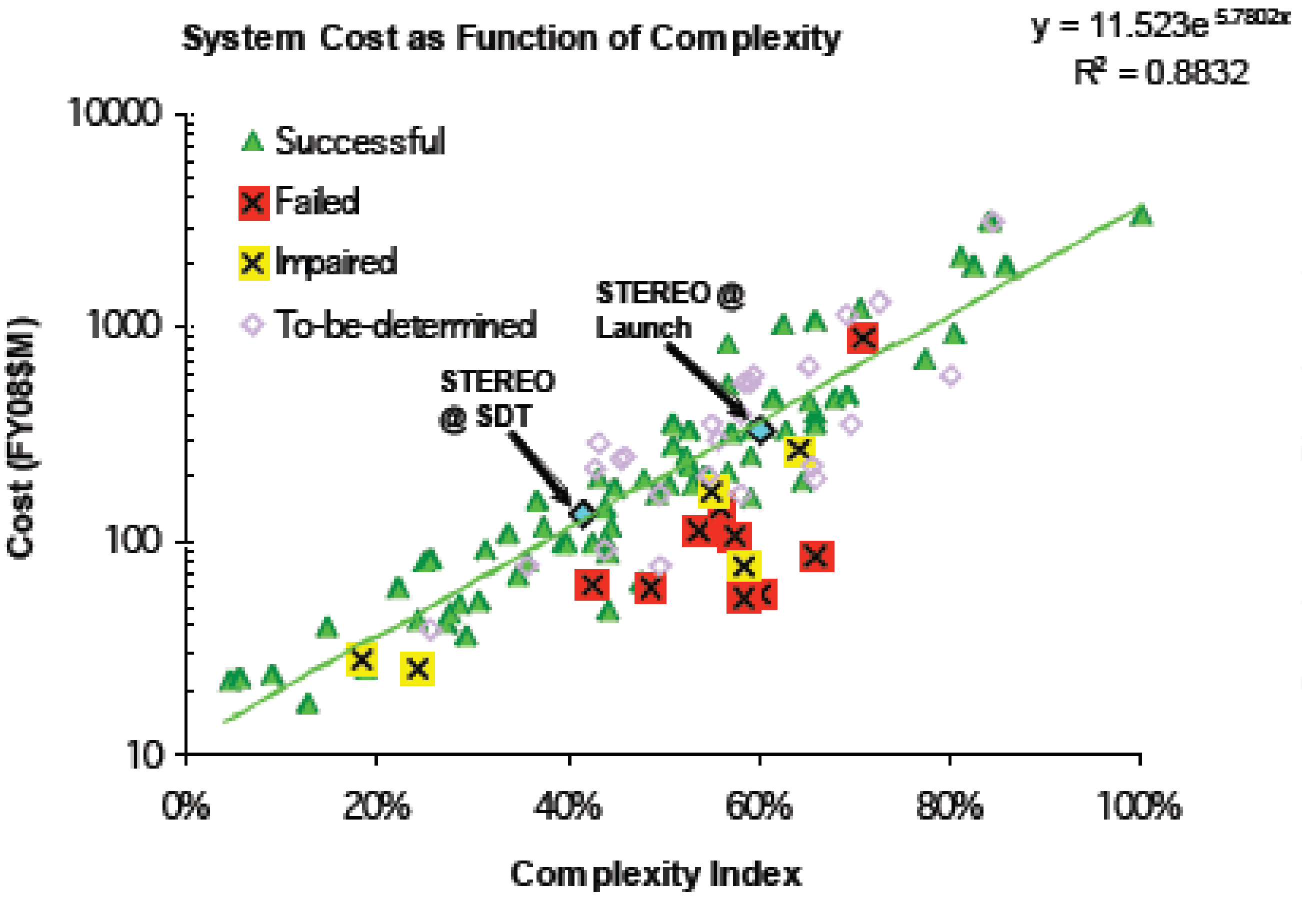
Complexity increase +42%

	C_1		C_2		C_3		C		C/C_{ML}		C_{new}/C_{old}
	Old	**New**	**Old**	**New**	**Old**	**New**	**Old**	**New**	**Old**	**New**	
Most Likely	161	188	126	184	1.51	1.69	351	499	1	1	1.42
Mean	179	244	141	240.4	1.51	1.69	392	650.3	1.12	1.30	1.65
Median	178	242	139	238.9	1.51	1.69	388	646.8	1.10	1.29	1.66
70 percentile	181	247.9	145	246.2	1.51	1.69	399.6	663.94	1.14	1.33	1.66

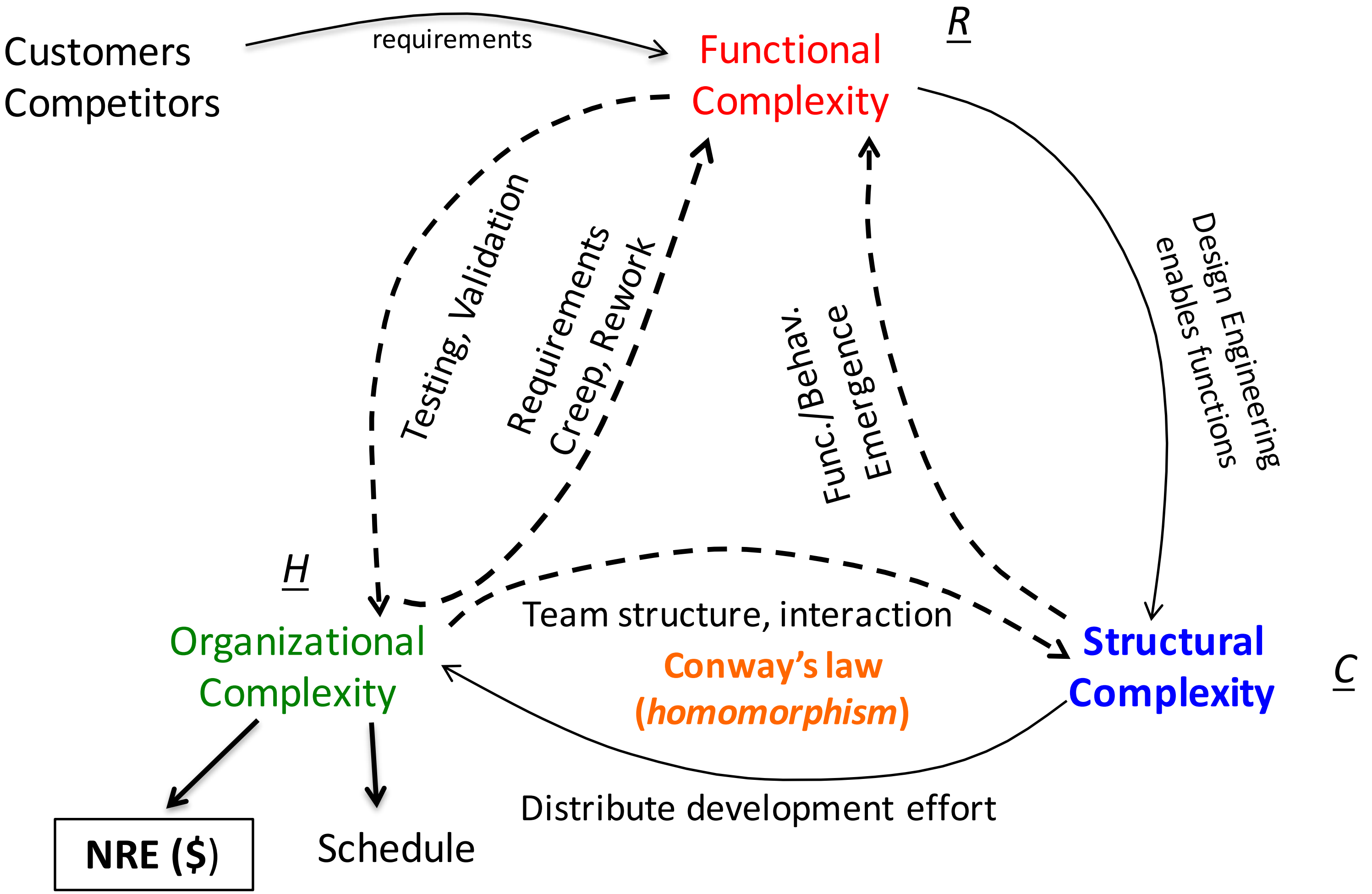
- Trend towards more distributed architecture with higher structural complexity and significantly higher development cost*. Similar trend was observed in Printing Systems.

Development Cost (NRE) and "Complexity"

- **CoBRA (Aerospace Corp., 2008) – Complexity Index based on analysis of historical data.**
- **Projects that were highly complex but tried to cut development cost had high failure rates**



Three Dimensions of Complexity



NRE Cost – Non-Recurrent Engineering Cost

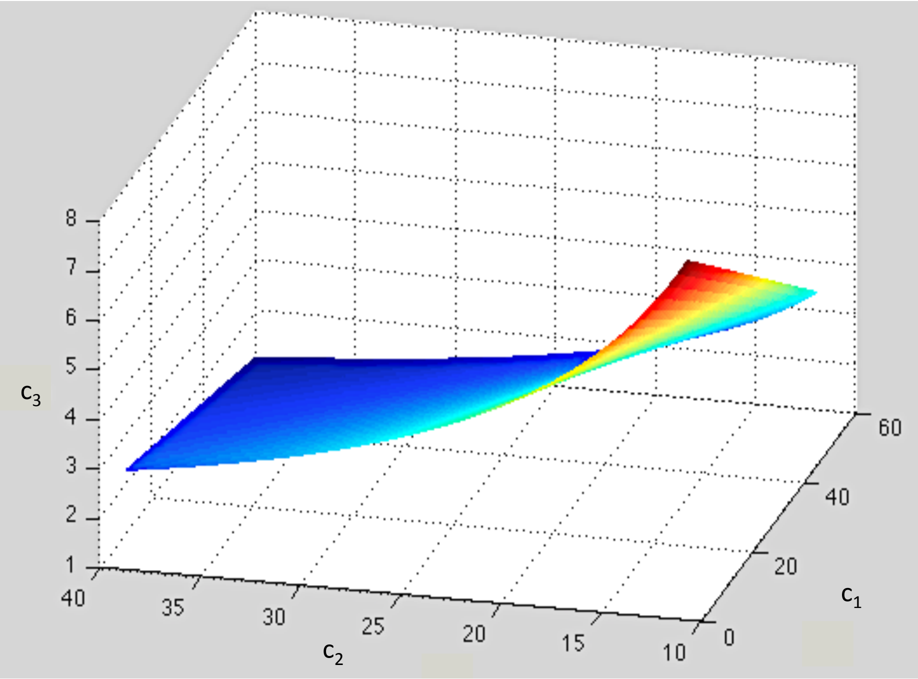
The First Law of Systems

- First Law of Thermodynamics (ca. 1850):
  - The law of conservation of energy states that the total energy of an isolated system is constant; energy can be transformed from one form to another, but cannot be created or destroyed.
    - $E_{kin} + E_{pot} - U = 0$

- The First Law of Systems Engineering (ca. 2013)
  - Given a fixed set of functional requirements *R* and human organizational architecture *H*, the total complexity of a system, *C* , is conserved. Complexity can be traded between its components and its interfaces and topology but cannot be decreased beyond a minimum level.
    - $C_1 + C_2 * C_3 - R - H = 0$

Implication 1: Iso-Complexity Tradeoffs

- Once we define the level of complexity, there are different ways to distribute this total structural complexity, C into its three constituents $\{C_1, C_2, C_3\}$: *IsoComplexity Surface*

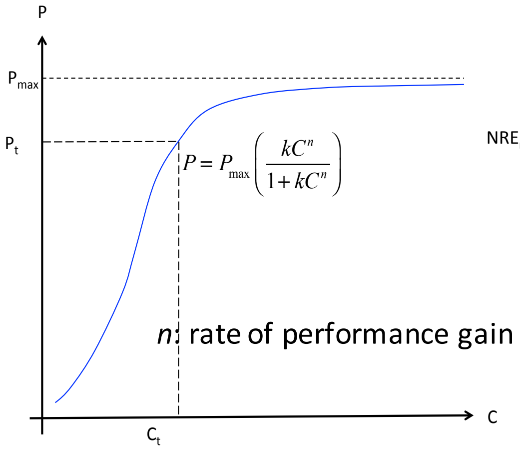


Iso-complexity surface: n = 20 components, assuming, c_1 in [10,60]; c_2 in [12,40] and C = 100.

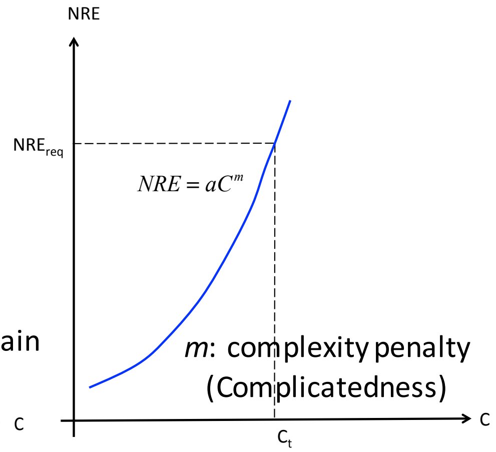
- Tradeoff between (i) complex components and simple architecture, or (ii) simpler components and more complex architecture.
- Choice can be made depending on complexity handling capabilities of the development organization. E.g.
  - Excellent component designers
  - Skilled Systems integrators
  - Etc …

Implication 2: Need for Complexity-based Budgeting

Complexity budget is the level of complexity that maximizes Value !



n: rate of performance gain



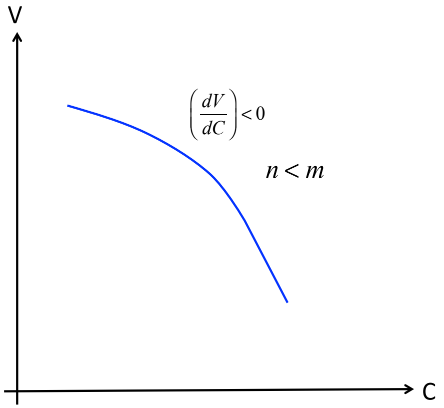
m: complexity penalty (Complicatedness)

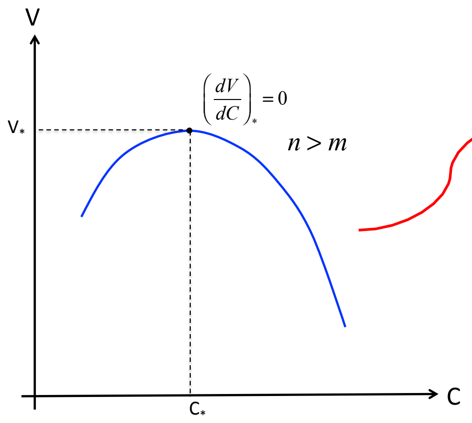
$$P = P_{max}\left(\frac{kC^n}{1+kC^n}\right)$$

$$NRE = aC^m$$

$$\boxed{V = \frac{P}{NRE} = P_{max}\left(\frac{k}{a}\right)\left[\frac{C^{(n-m)}}{1+kC^n}\right] = S\left[\frac{C^{(n-m)}}{1+kC^n}\right]}$$

Value function as the complexity price for performance gain – Maximize V:





$$C_*^n = \frac{\left(\frac{n}{m}\right)-1}{k}; \quad P_* = P_{max}\left(1-\frac{m}{n}\right)$$

$$NRE_* = a\left[\frac{\left(\frac{n}{m}\right)-1}{k}\right]^{\frac{m}{n}}; \quad V_* = S\left(\frac{m}{n}\right)\left[\frac{\left(\frac{n}{m}\right)-1}{k}\right]^{\left(1-\frac{m}{n}\right)}$$

Summary of key points

- Structural complexity of man-made systems has been increasing steadily since the industrial revolution
- This is driven by customer needs and competition → functional complexity R → structural complexity C → organizational complexity H
- A rigorous measure of complexity is based on graph energy of the DSM
  - Satisfies Weyuker's criteria (1998)
  - C = C1+ C2*C3;
  - C3: Graph Energy is a measure of topological complexity
  - Iso-complexity based budgeting with clear targets is needed
- **First Law of Systems Engineering (according to Sinha-de Weck): Conservation of Complexity**
  - **Given a set of functional requirements R, calculate minimum needed structural complexity C, and adjust for organizational complexity H to satisfy the first law**
- Violating the first law can lead to project or system failure !

Questions?   Comments?